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The beta heuristic from a time/frequency perspective: A wavelet analysis of the market risk of sectors[☆]

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ABSTRACT

Wavelet methodology is used to estimate scale betas for eleven industry/sectors for the period 1986-2016. A comparison of scale betas with standard regression estimates of betas finds no significant differences for any of the sectors at high frequency/low scales. However, for most of the sectors there are significant differences at medium and high scales. A rolling 60 month window shows that scale betas may differ from standard betas substantially for several years. Implications for portfolio managers, especially those employing beta rotation strategies, are provided.

1. Introduction

The market exposure of an investment is a well-recognized source of risk that portfolio managers must take into account. The Capital Asset Pricing Model (CAPM) developed by Sharpe (1964), Linter (1965), and Mossin (1966) continued the path breaking research of Markowitz (1952) on the risk reducing effects of portfolio diversification by introducing beta, a measure of systematic risk that captures the non-diversifiable risk of an investment. The degree that market exposure captured by beta does the job of assessing risk well has been subject to a great deal of research. Whether or not a consensus regarding the best approach to defining and estimating market risk is reached, for portfolio managers beta is a fact of life. For this reason, we view the widespread use of beta as a measure of an investment's systematic or non-diversifiable risk by investment managers similar to that of a decision-making heuristic.[1] In this case, a short-cut method for understanding and comparing market risk across investments. As Bollerslev et al. (2016) comments, “Even though numerous studies over the past half-century have called into question the ability of the capital asset pricing model(CAPM) to fully explain the cross-section of expected stock returns, the beta of an asset arguably remains the most commonly used systematic risk measure in financial practice.”[2] Another fact of life for portfolio managers is the existence of investments in sectors done relatively cheaply through ETFs or mutual funds.[3,4] Khorana and Nelling (1997) find that the most important factor explaining variation in sector-fund returns is the return on the market index.

The presence of both short and long-term market participants is another fact of life for the investing world. Reconciling this fact with estimates of beta is not something that at least on the intuitive level one would think that standard CAPM regression-based estimates of beta does well. This is because the standard market beta is based on

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[1] There is a large literature on heuristics in decision making. An excellent overview is found in Kahneman (2011).
[2] p. 464
[3] Based on data from the Investment Company Institute (May 2016) 334,965 million is invested in U.S. sector/industry ETFs. A recent IMF report on the Asset Management Industry (2015) provides evidence of the growth of focused investments.
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assumptions that place restrictions on time horizons and frequency changes.[5]

Wavelet methods have gained widespread acceptance as an efficient means of investigating multi-horizon properties of time series. Wavelets provide a unified framework for investigating the relationship among variables across frequencies and over time.[6] Recent research that offers empirical support for time-scale differences among investors in equity and commodity markets is found in Vacha and Barunik (2012), Aloni et al.(2013), Bekiros and Marcillino (2013), Graham et al. (2013), and Bekiros et al. (2016). Rua and Nunes (2012) use wavelet methodology and provide evidence that market risk varies across time and over frequencies.[7] Huang and Hueng(2008) estimate a time-varying beta model applied to the ten S & P 500 sectors, but do not consider time-varying behavior at different frequencies.[8] Our paper differs from previous research in that we investigate the market risk of sectors through the use of a well-accepted methodology for dealing with time-scale differences.

In this paper, we estimate betas for eleven market sectors using wavelet analysis and compare wavelet betas with standard regression-based betas. One result is that for all eleven sectors, low scale betas (2-4 months) are not significantly different from standard betas. However, when it comes to medium and high scale betas (we use six scales that range from 2-128 months, where the highest frequency or lowest scale is 2-4 months) we find a different story in that all but one of the sectors has at least one scale beta significantly different from the standard. As our analysis illustrates the differences that are found to be significant at medium and high scales vary depending on the sector and scale. The one sector without any significant differences between scale and standard beta is a high beta sector, Business Equipment. In our analysis whether or not there are significant differences between standard regression estimates of beta and scale betas appears consistent with the story told by the wavelet coherence plot (wavelet coherence is a measure like correlation, but localized in time-scale space and not limited to linear dependencies). In the case of Business Equipment, as our coherence plot illustrates there are no breaks in coherence even over medium and high scales.

Although there are a number of reasons to estimate beta coefficients for sectors with a methodology that captures multi-period investment horizons, we find that at low scales it does not matter, while for most sectors at medium and high scales it does. For portfolio managers our results can be used to turn the beta dial in a direction that helps improve its use. For example, portfolio managers who use beta rotation strategies that rely on low beta sectors as protection against market downturns should use scale betas that reflect horizon effects. As our rolling window estimates of scale betas at medium scales illustrate for widely recognized low beta sectors such as utilities and health, time-scale considerations have significant effects on beta estimates.[9] We also find sectors such as Telecom that switch from a low beta category using the standard beta estimate to a high beta category based on scale betas estimated at higher scales, and a sector, Manuf, that switches from a high to low beta category.

While it is not controversial to assert that supply and demand shocks impact sectors at different times or horizons, this fact is not sufficient for generating low scale betas that are significantly different from standard estimates. We are applying wavelet methodology, a methodology that captures horizon effects applied to a context where there exist factors driving sector returns that work over multiple horizons, but do not find significant differences between scale and wavelet betas at low scales. Wavelet methodology captures unique information at each horizon, and we surmise that high frequency changes are not contributing to market risk at the sector level perhaps because high frequency changes represent short-lived shocks that are more likely to reflect diversifiable risk that is not captured in estimates of beta coefficients. This is not the case at high and medium scales. The medium and high scale dynamics play out differently in that for ten of the eleven sectors there is at least one and as many as four scale dependent betas that are significantly different from the standard estimates. We argue that there are changes in the market environment occurring at medium and high scales that differ in important ways from changes at low scales. However, the time and frequency changes occurring at medium and low scales are not all created equal. Some changes do lead to scale dependent betas that are significantly different from standard estimates, but not every scale beta at medium or high scales is significantly different. We relate this to whether there are coherence differences across medium and high frequencies.

Differences in estimates of scale betas and standard betas across sectors are also compatible with Siegel (2005) where he argues that the diffusion of market moving information within sectors and across sectors is uneven. His explanation is compatible with our results since such unevenness may be captured by changes in coherence across frequency that wavelet analysis uncovers. Put differently, since wavelet analysis captures changes in the frequency domain over time we are able to identify periods or scales when estimates of the systematic risk of sectors are significantly changed relative to standard estimates of beta. Wavelet measures of market betas for the sectors provide significantly different measures of market betas estimated from the standard one-factor market model when the frequency resolution of low frequencies and the time resolution of high frequencies are important features of the underlying risk dynamics. We find this occurs when there are large differences in coherence across frequencies. Our finding that the standard market beta of the business equipment sector for the period examined (1986-2016) is not significantly different from scale betas even at medium and high scales is explained by its high, but stable coherence over the period examined.

This paper employs a data set that includes the following periods of high market volatility: 1) Asian Crisis of 97-98, 2) tech bubble burst of 2000, 3) financial crisis of 2008-2009, and 4) the European debt crisis (2010-2011).[10] Our analysis of the data also highlights through results from a Multiresolution Analysis that periods of market turmoil are associated with high market volatility at low scales, but only the financial crisis of 2008-2009, followed by the European Debt Crisis is associated with periods of high market volatility at high scales. The presence of high market volatility at high scales we refer to as a ”market turn,” while a ”market shrug” refers to high market volatility at low scales. An examination of the wavelet power spectrum for the market and each sector illustrates that the pattern of variation among sector returns even during periods of market turmoil, appears differently at different time horizons and frequency intervals. Some of these periods are market shrugs affecting few sectors, while market turns are felt across many sectors over many different scales.

The remainder of the paper is organized as follows: Section 2 highlights research based on wavelet analysis in applied financial economics of particular relevance for our analysis. The important concepts used in wavelet analysis that are applied in our analysis are

[5] A voluminous literature devoted to empirical tests of the CAPM evolved. Much of the empirical work on the CAPM employs a beta that remains constant over time or over the estimation period. One fix for this is found in time-series variation in the conditional betas of equity portfolios as shown in research by Bollerslev et al. (1988). Recent research by Bali (2008) have expanded the seminal inter-temporal capital asset pricing model (ICAPM) found in Merton (1973) using novel econometric techniques.

[6] Wavelet methodology has been employed across research fields, with growing applications in economics and finance, see Conlon and Cotter (2011). Research on wavelet methodology of particular relevance for our paper is discussed in the next section.

[7] Their application is to Emerging Markets.

[8] Their focus is on the asymmetric risk-return relationship and they do not employ wavelet analysis.

[9] See ?Business Cycle Approach to Equity Investing? by Fidelity Investments(2014).

[10] These periods of high volatility have been identified as periods of crisis in such research as Bekiros et al. (2016).

introduced in Section 3. The data and empirical results are discussed in Section 4. The conclusions follow in Section 5.

2. Literature review

The role of beta as a measure of market risk is given center stage in the CAPM developed by Sharpe (1964), Linter (1965) and Mossin (1966). The long history of empirical studies that followed their seminal work is summarized in Fama and Kenneth (2004). One important issue emerging from empirical testing of the CAPM is the stability of beta over time. Attempts to improve estimates of beta resulted in the removal of the restriction that beta remain constant over the estimation period. An important early study that allowed for time-series variation in the conditional betas of equity portfolios is Bollerslev et al. (1988). Subsequent research by Harvey (1989); Jagannathan and Wang (1996); Lewellen and Nagel (2006); Bali (2008); Bali and Robert (2010), and Bali and Robert (2012) found significant time-series variation in conditional beta. Recent research by Bali et al. (2016) allows for time-varying sensitivity of an asset to the market portfolio and to shifts in future investment opportunities. Another path for research that is not beholden to a constant beta assumption employs the wavelet methodology. This approach allows beta to vary over frequency and time. In doing so it allows one to examine the effects of heterogeneity in investment horizons. Wavelets models are not constrained to two scales, the “short term” and “long term”. Gencay et al. (2003) first proposed the use of wavelets to estimate systematic risk in the Capital Asset Pricing Model. They estimate the beta of each stock annually for 6 wavelet scales using daily returns for the period January 1973 to November 2000 for stocks that were in the S & P 500.[11] They find a positive relationship between portfolio returns and beta. Gencay et al. (2005) extend their 2003 study by including stocks from the Germany, and UK. They find that scale matters in other markets in that the relationship between portfolio returns and beta becomes stronger at high scales. Fernandez (2005) applies wavelet analysis to a model of the international CAPM using a data set that consists of daily aggregate equity returns for seven emerging markets for the period 1990-2004.[12] The ICAPM[13] was estimated at 6 scales (2-128 day dynamics). Fernandez finds that market sensitivities are generally greatest at the higher scales of 5 and 6. In addition, the R^2 peaked at scales 5 and 6. She concludes that the ICAPM does its best at capturing the relationship between risk and return at the medium scale or long term scale that for their data set is 32-128 days. An important takeaway from research employing wavelet measures of beta is that when the environment is distinguished by slowly changing features, or low frequency events the CAPM's applicability in terms of providing a measure of systematic risk improves when using wavelets. This is consistent with the findings of Rua and Nunes (2012) that employs wavelet methodology and provides evidence that market risk varies across time and over frequencies.[14]

Another path of research relevant to our study of the market risk of sectors is research that examines the co-movement of returns using wavelet analysis. Rua and Nunes (2009) examine the international co-movement of returns using wavelet analysis. They use the continuous wavelet transform to study the coherence of monthly returns from January 1973 to December 2007 for the US, Germany, Japan and the United Kingdom at the aggregate level and by industry sector. One of the main findings of their analysis is that the strength of the co-movement of returns across countries depends on the frequency domain. Specifically, they find that the co-movement is stronger at lower frequencies. Barunik et al. (2011) examine the co-movement of Central European stock markets using wavelet coherence on high frequency data. Their data set consists of TICK data of stock indices for Germany, Czech, Poland and Hungary from January 2, 2008 through November 30, 2009 sampled in five-minute intervals. They also examine co-movement among the indices at the daily level and include the U.S. and the U.K. in their analysis. For both data sets they find that co-movement of stock markets changes significantly in time and varies across scale.[15] Recent research that offers empirical support that time-scale differences among investors matters in both equity and commodity markets is found in Vacha and Barunik (2012), Aloni et al(2013), Bekiros and Marcellino (2013); Graham et al. (2013), and Bekiros and Nguyen (2016). A wavelet approach for estimating market risk has distinct advantages for capturing changes in the frequency domain over time. However, the literature reveals a gap. Wavelet estimates of beta did not consider sectors. Sectors are of particular importance since many active investors employ portfolio tilting techniques which utilize sectors portfolios throughout the course of a business cycle. Huang and James Hueng (2008) estimate a time-varying beta model applied to the S & P 500 sectors, their estimates do not consider time-varying behavior at different frequencies. Our research is the first to offer estimates of scale betas based on sectors. The research literature points to the importance of capturing differences in investor horizons[16], and changes in the frequency domain in estimates of market risk, as well as, for measuring return co-movements. Our application of wavelet methodology to the market risk of sectors suggests that when it comes to capturing the market risk of sector investing a nuanced perspective is needed in that no simple patterns emerge.

3. Wavelet analysis

The main feature of wavelet analysis that has broadened its applicability in finance is its capability to decompose a time series into low and high frequency components that correspond to short, medium and long term variation in the series. Both time and frequency components of a series are captured through wavelets that represent a set of basis functions that are classified into father and mother wavelets. The father wavelet captures smooth and low frequency components, while the mother wavelets capture the short-term dynamics or high frequency parts.[17] In contrast to Fourier methods where the basic Fourier transform frequency decomposition is global, the wavelet transform allows for localized decomposition in both frequency and time. This is particularly suitable for an analysis where there are investors with different time horizons.[18]

The transformation is not in terms of trigonometric polynomials, but in terms of wavelets.[19] The wavelet transform is composed of a father wavelet and a set of mother wavelets. Given a function Φ, the father wavelet for the discrete transform is defined as:

$$\Phi_{J,k} 2^{-\frac{J}{2}} \Phi \frac{t-2^{J} * k}{2^{J}} \tag{2}$$

$$\int \Phi(t)\, dt = 1$$

The mother wavelets, also in discrete form, are defined as:

[11] The calculation of scales is described in Section 3.

[12] Brazil, Chile, Mexico, Indonesia, South Korea, Malaysia, and Thailand.

[13] ICAPM for 2 countries $E(r_i - r) = \beta_1 cov(r_i, r_w) + \beta_2 cov(r_i, s)$, where r_i=returns for domestic asset, r_w=returns for world portfolio, s is the percent change in the exchange rate for domestic and foreign currency.

[14] Their application is to Emerging Markets.

[15] Applications of wavelet methodology in finance and economics are growing rapidly. See for example, Conlon and Cotter (2011), Fernandez and Ratios (2007); Kriechbaumer et al. (2014); Wang and Wu (2012).

[16] Earlier studies include Levhari and Levy (1977) and Handa et al. (1989), see also Jagannathan and Wang (2007)

[17] See Cowley (2005) for an introduction to wavelet methods in economics and finance, and Gencay, et. al. An Introduction to Wavelets and Other Filtering Methods in Finance and Economics.

[18] For the relevance of horizon effects see for example Kamara et al. (2015).

[19] See Strong (1993) for a comparison of wavelet versus Fourier transforms.

$$\Psi_{j,k}\ 2^{-\frac{j}{2}}\Psi\frac{t-2^{j}*k}{2^{j}},\ j=1,\ldots,J \tag{3}$$

$$\int\Psi(t)dt=0$$

Where J is the number of scales or levels, 2^{J} is a scale factor and k is the time domain index.

The father and mother wavelets are each indexed by both scale and time. It is precisely this dual indexing that makes wavelet analysis appealing since as a time series, f(t), is represented as a linear combination of wavelet functions that are localized in space and time.

The scale parameter is inversely proportional to frequency.[20] The father and mother wavelet functions may also be represented as filters. In this alternative representation the father wavelet is a low pass filter, and the mother wavelets are high pass filters.[21] We can use the wavelet functions to transform a time series, f(t), into a series of wavelet coefficients,

$$S_{J,k}=\int f(t)\Phi_{J,k} \tag{4}$$

and,

$$d_{j,k}=\int f(t)\Psi_{j,k}\quad j=1,\ldots,J \tag{5}$$

Where $S_{J,k}$ are the coefficients for the father wavelet at the maximal scale, 2^{J}, and the $d_{j,k}$, are the coefficients of the mother wavelets at the scales from 1 to 2^{J}. The $d_{j,k}$ are referred to as the detailed coefficients and the $s_{J,k}$ are referred to as the smooth coefficients. Applying the transforms results in a time series of length k of smooth coefficients at the maximal scale J, and J time series of detailed coefficients each of length k. If there are 6 scales, the frequency of the first scale is associated with the interval [1/4,1/2], and the frequency of scale 6 is associated with the interval [1/128, 1/64]. The time series used in this a paper is monthly. We decompose the series into six scales (D1-D6) that correspond to 2-4, 4-8, 8-16, 16-32, 32-64, and 64-128 months. The smooth component (S6) captures the trend of the original series. The high frequency component is associated with the shortest scale D1, while the low frequency component is associated with the longest scale D6. The use of six scales is found in much research the employs wavelet methodology in economics and finance. For example, see Gencay et al (2002), Rua and Nunes (2009), and Bekiros(2015). Six scales are considered to provide a good balance between time and frequency localization.

Our application of wavelet analysis to sectors employs both a discrete wavelet transform (DWT) and a multi-resolution decomposition. Given the smooth and detailed coefficients, a time series f(t) can be represented in decomposed form, known as the multi-resolution analysis of f(t), as follows:

$$f(t)=\Sigma_{k}S_{J,k}\ \Phi_{J,k}\ (t)+\Sigma_{k}d_{J},\ k\Psi_{J,k}\ (t)+\cdots+\Sigma_{k}d_{j,k}\ \Psi_{j,k}\ (t)+\cdots+\Sigma_{k}d_{1,k}\ \Psi_{1,k}\ (t) \tag{6}$$

Or, using summary notation,

$$f(t)=S_{J}+D_{J}+D_{J-1}+\cdots+D_{1}$$

The discrete wavelet transform decomposes a time series into orthogonal signal components at different scales. S_j is a smooth signal, and each D_j is a signal of higher detail. The number of coefficients differs by scale. If the length of the data series is n, and divisible by 2^{J}, there are $n/2^{j}d_{j,k}$ coefficients at scale j=1,...,J-1. At the coarsest scale there are $n/2^{J}d_{J,k}$ and $s_{J,k}$ coefficients. The wavelet variance at each scale is captured as the wavelet power of each scale. The continuous wavelet transform (CWT) is also useful for gaining insight into the time-scale characteristics of a time series. The CWT is defined as,

[20] See Gencay et al., 2010, pp. 99-103 for a complete discussion.

[21] See Ramsey (2002).

$$W(\lambda,t)=\int_{-\infty}^{+\infty}\Psi_{\lambda,t}\ (u)x(u)du \tag{7}$$

where,

$$\Psi_{\lambda,t}\ (u)\equiv\frac{1}{\sqrt{\lambda}}\Psi\left(\frac{u-t}{\lambda}\right)$$

As noted by Ramsey, the main difference between the CWT and DWT is that the CWT considers continuous variations in the scale (λ) and time components (t). The discrete wavelet transform can be derived independently of the CWT, but it can also be viewed as a critical sampling of the CWT with $\lambda=2^{-j}$ and $t=k2^{-j}$.

The wavelet power spectrum which measures the local variance of a time series at different scales is defined as $|W(\lambda,t)^{2}|$, and aids our analysis in terms of understanding how periodic components evolve over time when applied to the market, as well as, the eleven sectors examined in our analysis. A clear advantage that the CWT has over the discrete transform is that it produces a powerful visual for detecting time-scale patterns. The wavelet power spectrum is helpful for understanding how the power varies with the scaling of the wavelet. But we also need to understand how periodic components evolve jointly over time. The Fourier coherency identifies frequency bands where two time series are related, while the wavelet coherency identifies both frequency bands and time intervals when time series are related. The wavelet coherence of two series, x and y, is a measure of co-movement across time and scale based on the CWT. To define it we need the definition of two other measures, the cross wavelet transform (XWT) and the cross wavelet power (XWP). The XWT is defined as

$$W_{xy}=W_{x}(\lambda,t)W_{y*}(\lambda,t) \tag{8}$$

The XWP is the defined as the absolute value of the XWT, $|W_{xy}(\lambda,t)|$. It measures the local covariance of x and y at different time scales. The XWP identifies areas in time-scale space where the two series have high common power. In addition to identifying the common power of two time series, we are also interested in identifying areas of co-movement in time-scale space, even if the cross wavelet power is low. A measure of co-movement, the wavelet coherence, is defined as:

$$R^{2}(\lambda,t)=\frac{|S(S^{-1}W_{xy}(\lambda,t))|^{2}}{S(S^{-1}|W_{x}(\lambda,t)|^{2})*S(S^{-1}|W_{xy}(\lambda,t)|^{2})} \tag{9}$$

Where S is a smoothing operator in time and scale, and $0\le R^{2}(\lambda,t)\ge 1$. The wavelet coherence is similar to the correlation coefficient, and is typically interpreted as a localized correlation in time-scale space.

4. Data and empirical results

4.1. Cumulative excess returns

The data we use for our analysis are from the Kenneth French Data Library.[22] The market portfolio (MKT) is a composite portfolio of all stocks traded on the NYSE, AMEX, and NASDAQ. The market is divided into 12 industry groups or sectors defined below. The Other Industry is not included in our analysis as it is essentially a residual category and is not relevant from an investment strategy perspective. We use the abbreviation associated with each sector throughout the paper.

All returns are reported in excess of the risk free rate. The risk-free rate is measured by the yield on the 1-month T-bill.[23]

The sample period includes three recessions: 1) July 1990 - March 1991 (8 months), 2) March 2001 - November 2001 (8 months), and the most recent recession, December 2007 - June 2009 (18 months).[24] As

[22] http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

[23] The 1 month T-bill rate used as a risk free rate is calculated by Ibbotson and Associates, and provided by Kenneth French in his Data Library

[24] Recession dates are from, US Business Cycle Expansions and Contractions, the National Bureau of Economic Research. http://www.nber.org/cycles.html

Table 1
Kenneth French 12 Industry Data Set.

1	NoDur Consumer NonDurables – Food, Tobacco, Textiles, Apparel, Leather, Toys
2	Durbl Consumer Durables – Cars, TV's, Furniture, Household Appliances
3	Manuf Manufacturing – Machinery, Trucks, Planes, Off Furn, Paper, Com Printing
4	Enrgy Oil, Gas, and Coal Extraction and Products
5	Chems Chemicals and Allied Products
6	BusEq Business Equipment – Computers, Software, and Electronic Equipment
7	Telcm Telephone and Television Transmission
9	Shops Wholesale, Retail, and Some Services (Laundries, Repair Shops)
10	Hlth Healthcare, Medical Equipment, and Drugs
11	Money Finance
12	Other Other – Mines, Constr, BldMt, Trans, Hotels, Bus Serv, Entertainment

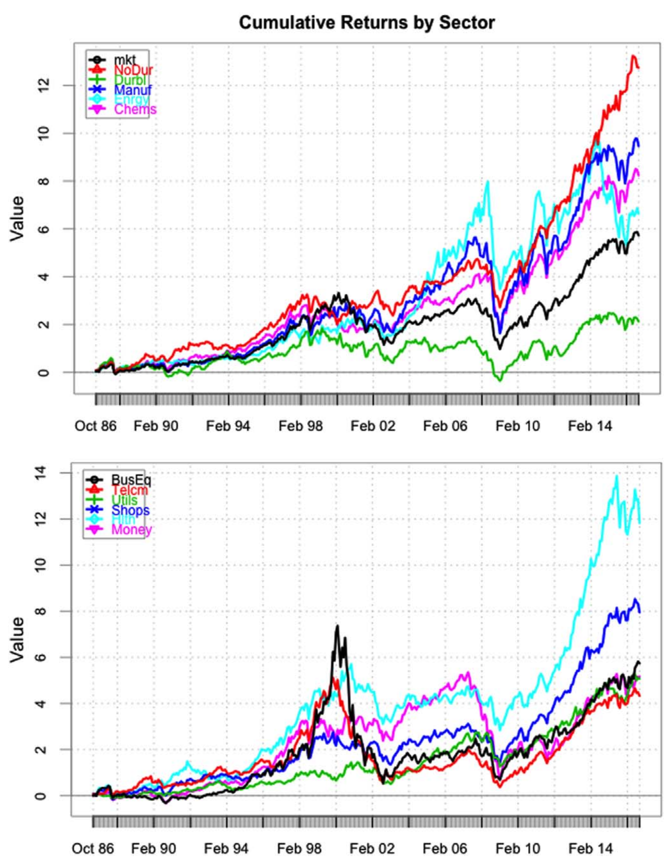

Fig. 1. These figures show the time series of cumulative excess returns for the Market Index, and the eleven sector indices based on monthly data from October 1986 through October 2016.

well as, periods of high market volatility, or market turmoil such as the Asian crisis of 1997-1998, and the European debt crisis of 2010-2011. Table 1.

Cumulative excess returns for the series are displayed in Fig. 1, and reported in Table 2. Cumulative growth for the MKT was 573% from October 1986 through October 2016. Those sectors with the highest cumulative excess growth are NoDur (+1274%), Hlth (+1183%), and Manuf (+946%). Cumulative returns for all three of these sectors were consistently at the high end of the sector distribution throughout the sample period. Other fast growing sectors such as Money and BusEq dropped substantially in a recession and never regained their lead. In the case of BusEq the drop occurred in the 2001 recession, and for the Money sector it occurred in the 2008 recession.

Two sectors with the lowest cumulative growth were Durbl (+212%) and Telcm (+433%). Telecommunications never regained the losses from the tech bubble burst, and the Durable goods sector was consistently low throughout the sample period.

4.2. Variability of sector returns

Our analysis of the sectors finds a high degree of variability in cumulative returns for some series, for example, Energy peaked at +784% in June 2008, and declined to +340% in Feb. 2009; Business Equipment peaked at 736% in Mar, 2000, and declined to +131% in Sept 2001; while Money peaked at +516% in July 2007 and declined to 66% in Feb. 2009. A more detailed set of descriptive statistics for the monthly excess returns (%) is reported in Table 2. Monthly returns range from a high of 42.6% for Durable goods (Apr. 2009) to a low of minus 42.8% also for Durable goods (Oct. 2008). Skewness is negative for all sectors except Durable goods; the skewness ranges from -0.873 for Manufacturing to +0.086 for Durables. Excess kurtosis is positive (leptokurtic) for all of the sectors, suggesting that the distribution of returns has fatter tails than a Normal distribution. It ranges from 0.7 for Utilities to 5.3 for Durables. The Jarque-Bera[25] test which tests the null hypothesis of Normality, rejects the null for all of the indices at the 95% level of confidence. The smallest JB test statistic is for Energy that has an asymptotic p-value of 0.0002.

Ljung-Box[26] statistics for market and sectors returns, and also the absolute value of returns are reported in Tables 3 and 4. Table 3 indicates that there is little evidence of serial correlation for returns. One exception is the durable goods sector where the null hypothesis is rejected at all 4 lag levels. Telecommunications rejects the null hypothesis at lag levels 6 and 12.

The Q-statistics in Table 4 are generally high enough to reject the null hypothesis, suggesting that presence of an ARCH effect. Two noticeable exceptions are Energy and Health. Both sectors fail to reject the null for the absolute value of returns at all 4 lag levels at the 95 % level of confidence.

In summary, the descriptive statistics, while typical of equity returns present a picture that suggests the need for a methodology that captures the underlying dynamics of a system characterized by equity returns at the sector level that exhibit non-normality, with a negative skew and positive excess kurtosis. For sector returns there is generally little or no serial dependence, but consistent with an ARCH effect, the absolute value of returns do exhibit serial dependence.

4.3. Multi-resolution analysis

Fig. 2 shows results from a multi-resolution analysis (MRA) of monthly returns for the market returns for the period from October 1986 through October 2016. The transformation is indexed by time and scale. The plot on the top left hand side (MKT) is the actual series of returns, the charts labeled D1-D6 are the wavelet details for $j = 2^J, j = 1, .., 6$. The smooth or father wavelet is plotted in the chart labeled S6. The scale for the chart is calculated as 2^j, so the period for the series labeled D1 is 2-4 months, D2 is 4-8 months, etc. As illustrated in Fig. 2, as j decreases from 6 to 1 the multiple resolution decomposition produces series of finer detail. Since the multi-resolution analysis is an additive decomposition, the original series is found by adding up at various time scales a detailed series up to the highest considered scale, and adding to this detailed series a smooth series that captures the long-term trend at the highest scale level, where the smooth series contains the non-stationary components if they exist.

The time evolution of the volatility components indicates that periods of high volatility are concentrated around specific times. Our data set covers the following periods of high market volatility: 1) The Asian Crisis of 1997-98, 2) tech bubble burst of 2000, 3) financial crisis

[25] The Jarque-Bera test statistic: $JB = \frac{n}{6}(S^2 + \frac{1}{4}(K-3)^2)$ where n=sample size, S=sample skewness and K=sample kurtosis. The JB statistic is distributed Chi-square with 2 degrees of freedom.

[26] The Ljung-Box Q statistic tests the null hypothesis that the data are random, H_0: $\rho(1) = \rho(2) = ? = \rho(j) = 0$. Critical values: for 1,6,12,and 24 df are: 3.84, 12.59, 21.03 & 36.42, respectively

Table 2
Summary statistics.

	Mkt	NoDur	Durbl	Manuf	Enrgy	Chems	BusEq	Telcm	Utils	Shops	Hlth	Money
Observations	361	361	361	361	361	361	361	361	361	361	361	361
Minimum	−0.232	−0.216	−0.327	−0.292	−0.189	−0.252	−0.265	−0.163	−0.127	−0.289	−0.211	−0.225
Median	0.012	0.008	0.007	0.013	0.008	0.011	0.011	0.011	0.010	0.009	0.010	0.012
Mean	0.006	0.008	0.006	0.008	0.007	0.007	0.008	0.006	0.006	0.007	0.008	0.007
Maximum	0.125	0.143	0.426	0.211	0.189	0.144	0.203	0.212	0.112	0.128	0.161	0.170
Stdev	0.044	0.040	0.069	0.054	0.054	0.045	0.070	0.051	0.039	0.049	0.046	0.055
Skewness	−0.903	−0.590	0.086	−0.873	−0.114	−0.660	−0.393	−0.365	−0.484	−0.780	−0.342	−0.706
Excess kurtosis	2.8	3.0	5.3	4.1	1.0	3.1	1.4	1.3	0.7	3.6	1.4	2.4
Jarque-Bera	173.4	159.0	436.3	298.6	17.0	177.8	39.2	34.2	21.7	238.4	37.8	117.5
Cum. Ret(%)	573.1	1274.4	212.7	946.4	664.0	824.2	574.1	432.8	506.9	794.9	1182.6	516.8

Table 3
Ljung-Box statistics- returns.

	Lag=1	Lag=6	Lag=12	Lag=24
Mkt	3.26	5.04	7.75	12.62
NoDur	1.90	7.11	15.33	30.61
Durbl	5.49	17.91	22.35	41.85
Manuf	2.14	8.96	12.64	19.25
Enrgy	0.01	5.23	11.14	14.53
Chems	0.53	2.89	6.56	13.96
BusEq	0.31	2.92	7.35	19.02
Telcm	1.78	12.60	22.32	30.14
Utils	0.57	3.55	12.35	19.03
Shops	5.19	14.13	17.00	24.80
Hlth	0.21	5.89	12.26	17.10
Money	5.28	13.59	18.35	29.25

Table 4
Ljung-Box statistics- absolute value of returns.

Absolute value of returns				
	Lag=1	Lag=6	Lag=12	Lag=24
Mkt	10.16	44.25	58.44	91.31
NoDur	**1.75**	24.11	39.81	59.36
Durbl	4.50	30.13	36.39	58.43
Manuf	7.49	36.66	48.60	67.27
Enrgy	0.93	3.61	19.98	25.15
Chems	3.20	10.21	21.84	33.92
BusEq	33.02	131.74	214.07	349.64
Telcm	10.19	68.48	97.27	141.27
Utils	0.00	18.64	33.05	53.78
Shops	7.53	32.28	41.64	57.61
Hlth	0.22	8.98	14.64	28.50
Money	15.96	77.97	91.04	103.30

of 2008-2009, and 4) European debt crisis of 2010-2011. These periods of market turmoil play out differently when viewed in terms of the detailed coefficients. At lowest scale, or highest frequency, each specific period of market turmoil is associated with high volatility, but only the financial crisis of 2008-2009, followed by the European debt crisis is associated with periods of high market volatility at low frequency or higher scales. While the series becomes smoother as the scale increases, we can see that at level 3 it appears as though the variance is greater after 2007. This same pattern of increasing variance post-2007 is also seen at levels D4 and D5.

The increasing variance after 2007 suggests that when it comes to market turmoil there is a market turn, and not a market shrug in that the downturn associated with 2008-2009 financial crisis was different in that it is associated with persistent market variability shown at higher scales. The international dimension of this crisis may help explain its persistence over scales.

4.4. Market and sector variability based on the wavelet power spectrum

The use of both scale and time in conceptualizing and understanding the variability of sectors and the market is illustrated in Fig. 3 that contains a plot of the power spectrum for monthly returns of the Market, October 1986-October 2016.

The level of the power is reflected by the color. The color intensity helps focus attention on how time and frequency are both incorporated into an understanding of variability. Blue represents the lowest variance and red the highest. The period, in months, is indicated on the vertical axis. If we take a given value for wavelet scaling, and read horizontally we see how the power varies across the time domain for a given scale. If at a given point in time we read vertically, we see the power varies with scale. The black contour lines indicate areas where the variance is statistically different from the variance of a white noise process.

The cone shaped edges that distinguish vivid colors from shaded colors is called the cone of influence. The power calculated outside the cone of influence violates the boundary conditions, and this implies that the power calculation can be influenced by edge effects, and should not be considered in the evaluation of power.

This graphic presents a similar story to the MRA as the variance is highest as shown in red after 2007. It clearly indicates a spike in the variance at the two-month period that centers around 2009, and also high variance during the same time frame at the 16-32 month scale. Although not appearing as a red color, we can see that the variance for the 32-64 month period from 1998 to 2012 appears to increase. There are significant spikes in variance at the 1-4 month period between 1998 and 2003, and again in 2008-2009, 2010-2011, reflecting short-run effects of periods with high market variability (Asian Crisis, Tech Bubble burst, Financial Crisis, and European debt crisis). When it comes to power, such periods appear to linger in the market as shown by the high intensity colors present at both low and high scales.

A striking feature of the analysis based on the wavelet power graph of returns for each sector is the wide variability of power across the time-scale space. The downturns in 2001 and 2008 both exhibit high power at a low period (2 months) for all 11 sectors. If we only considered the low period finding of high power the tech bubble burst in 2000 and the ensuing recession appear very similar to the housing bubble burst in 2008 and the ensuing recession in terms of the transmission across all eleven sectors in the short-run. However, when considering the 2008 downturn some sectors continue to exhibit high power over a broader range of periods (2-32 months). This is most notable for Energy, Money, and Durbl. The continued variability or power at higher scales for many sectors in 2010-2012 suggests that the financial crisis had lingering effects perhaps due to its transmission across borders. This spread of a recession is not evident in the 2001 recession. We do not find the high power at the higher scales (16-32

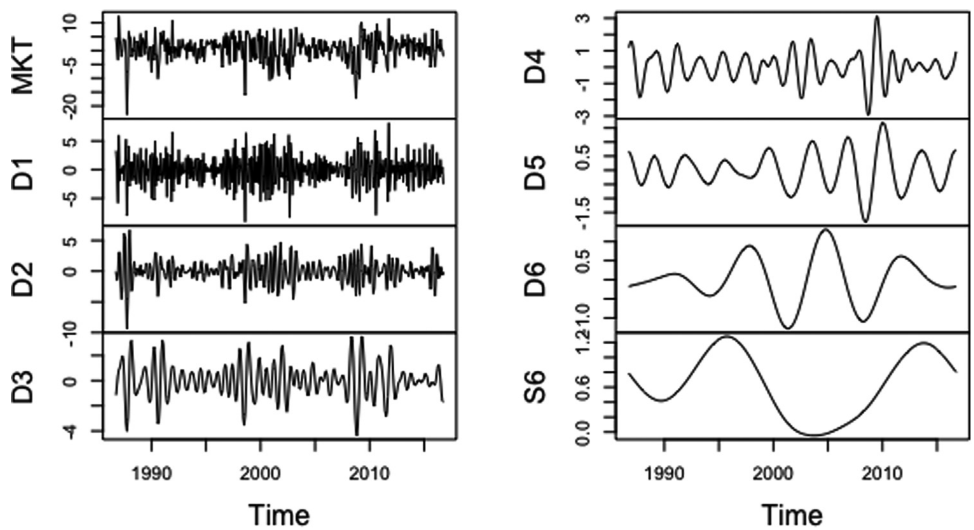

Fig. 2. Multiresolution analysis of Market returns This figure contains a multi-resolution analysis (MRA) of monthly returns for the total market for the period from October 1986 through October 2016. The series labeled MKT is the returns for the market Index. Series D1 through D6 are the wavelet details and S6 is the father wavelet.

month) across sectors that we find for the 2008 recession. However, the power spectrum for Telcom and BusEq is elevated over a broad range of periods during the tech bust (2000-2002).

Measured by power we find that although the short-run effects of the 2001 and 2008 recessions appear similar in terms of higher power for all eleven sectors, the long-run effects of the 2008 recession are more broadly based in terms of sectors compared with the 2001 recession. Two other sectors, Telcom and Utils, have very high power at a period of 32 months from 1998 through 2012. One interpretation is that during the sample period there were regulatory changes in both sectors and the high power at higher scales may reflect uncertainty regarding the effects of regulatory changes on sector performance and therefore, high power is picking up the effects of policy uncertainty in this sector.[27] In summary, the wavelet power spectra applied to the Market and eleven sectors clearly suggest the risk of sector investments does not present the same relationship at all frequencies. Whether the findings of high power are associated with significant differences in market risk for eleven sectors requires that we obtain a measure of how periodic components evolve over time, not just individually as reflected in the power spectrum for each series, but also jointly. In the next section, we examine the joint movement between market returns and sector returns by using wavelet coherence as a critical measure of co-movement. Fig. 4.

4.5. Sector analysis through wavelet coherency

The method of wavelet coherency is applied to the market and sector portfolios. Plots of the wavelet coherence are presented below (Fig. 5). In each plot x represents monthly excess returns for the market, and y represents excess returns for a sector. Arrows pointing to the right or left indicate that x and y are in or out of phase. An arrow pointing up (down) indicates that x (Market) leads (lags) y (sector). The color represents the level of the coherence or co-movement, where blue is the lowest and red is the highest. The white contours denote areas of significant (5% level) coherence. The presence of an arrow indicates that the wavelet coherence is statistically significant at the 95% level of confidence. The direction of the arrows indicates the phase of x, and y.

A striking feature of all eleven sectors is the low level of coherence with the market at high frequencies (from 1 to 4 months). This finding of low coherence with the market is present for less cyclical sectors, such as Utilites, Nondurables, and Health. These sectors have little or no coherence over time spans for periods of up to four months. We find that even for the more cyclical sectors the coherence is spotty for periods of less than 8 months. The coherence at high frequencies is surprisingly low, so we cross-check this result using traditional Fourier Spectral Analysis and find additional support for low coherence at high frequencies as shown in Fig. 6.

One possible explanation for the low level of coherence at high frequencies among all eleven sectors and the Market is that changes in short-term frequency components are short-lived and diversifiable.

The phase relationship found in the time-frequency plots of the continuous wavelet coherence suggest a complex lead/lag relationship. At the higher frequencies, the sectors are often in phase with the Market, but for lower frequencies this is typically not the case. The lead-lag relationship changes for many sectors across frequencies making it very difficult, and misleading, to characterize a sector as leading or lagging the market. However, the phase differences are telling the same story as the coherence charts in that for all the sectors with the exception of business equipment coherence differences are largest at medium and/or high scales.

Overall, the time-frequency plots of the continuous wavelet coherence show the time-varying nature of the relationship between sector and market returns at different frequencies. Even at high scales (low frequencies) there are periods where the coherency is large and other periods where it is small.

The time-frequency plots of continuous wavelet coherence paint a compelling picture of why capturing time-varying behavior at different frequencies is important for estimates of a sector's exposure to market risk. For example, the Utility sector (Utils) is a sector that has very little coherence with the Market for periods less than 8 months. It is clear from Fig. 5 that the coherence is high at low frequencies and low at high frequencies. Using a fixed time window for estimating market risk exposure may give a misleading picture of systematic risk because if the window is too short, it results in an underrepresentation of low frequencies components. If the window is too long (entire sample) then all frequencies are equally represented. To varying degrees the situation illustrated by Utils is true for most of the sectors, with the exception of Business Equipment. Business Equipment is a sector with large coherence that does not appear frequency dependent or time-varying. But for the other ten sectors the coherence of sector returns with the market is time-varying at different frequencies. We will investigate more formally whether such variation in coherence matters for estimates of market risk by first finding what we refer to as the baseline for market risk, standard beta estimates based on regressions of the market model. We also allow for time-variability through rolling

[27] For a discussion of regulatory changes affecting the Telecommunications see Noam (2006).

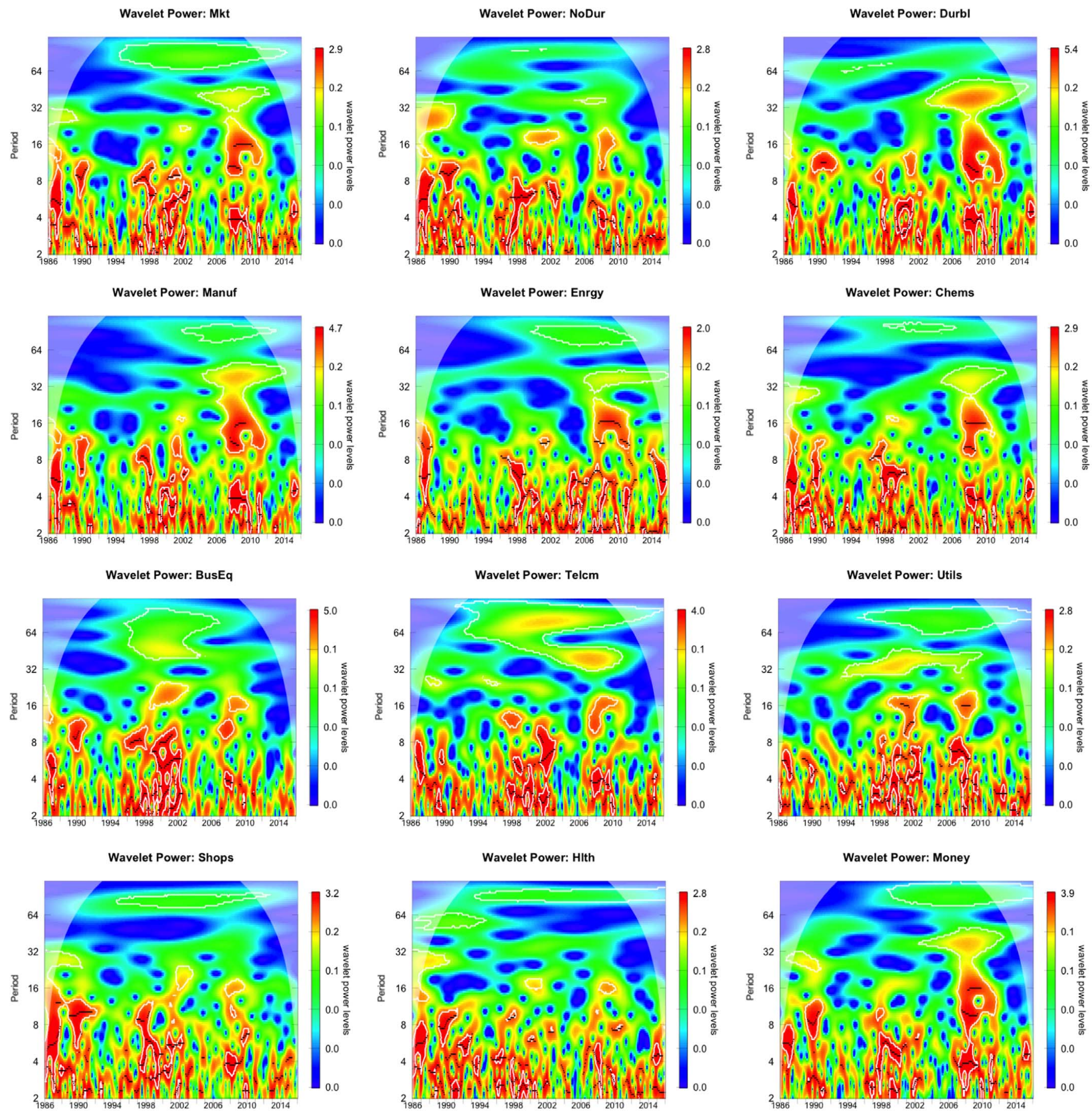

Fig. 3. These figures contain the wavelet power spectra for monthly returns of the total Market and each of the 11 Sectors for October 1986-October 2016.

window estimates of beta. Given that the strength of wavelet methodology is its ability to capture time-varying behavior at different frequencies (Fourier analysis only captures changes over frequencies) we also provide estimates of scale betas. To illustrate the importance of scale for estimates of beta, we also use a rolling time window to estimate time-varying betas at different scales.

4.6. The standard market model

We look at the data from the perspective of the standard market model in order to estimate a benchmark so commonly used, we argue it operates as a heuristic, and compare our estimates of this beta heuristic to estimates of wavelet betas.

In the static CAPM model investors care only about the one-period mean and variance of portfolio returns. We provide an estimation of the standard market model, where beta is estimated as an OLS regression coefficient.

$$r_{it} - r_{ft} = \alpha_i + \beta_i(r_{mt} - r_{ft})$$

where i=sector, r_f is the risk free rate, and r_m is the market return. The intercept or α_i is a measure of abnormal returns. In the context of CAPM we expect α_1 to be zero.

Parameter estimates for each of the 11 sectors are presented in Table 5. These estimates are based on the common practice of using monthly data for the entire sample period, for our analysis, Oct. 1986 to Oct. 2016. Hlth and Shops are the only sectors with intercepts that

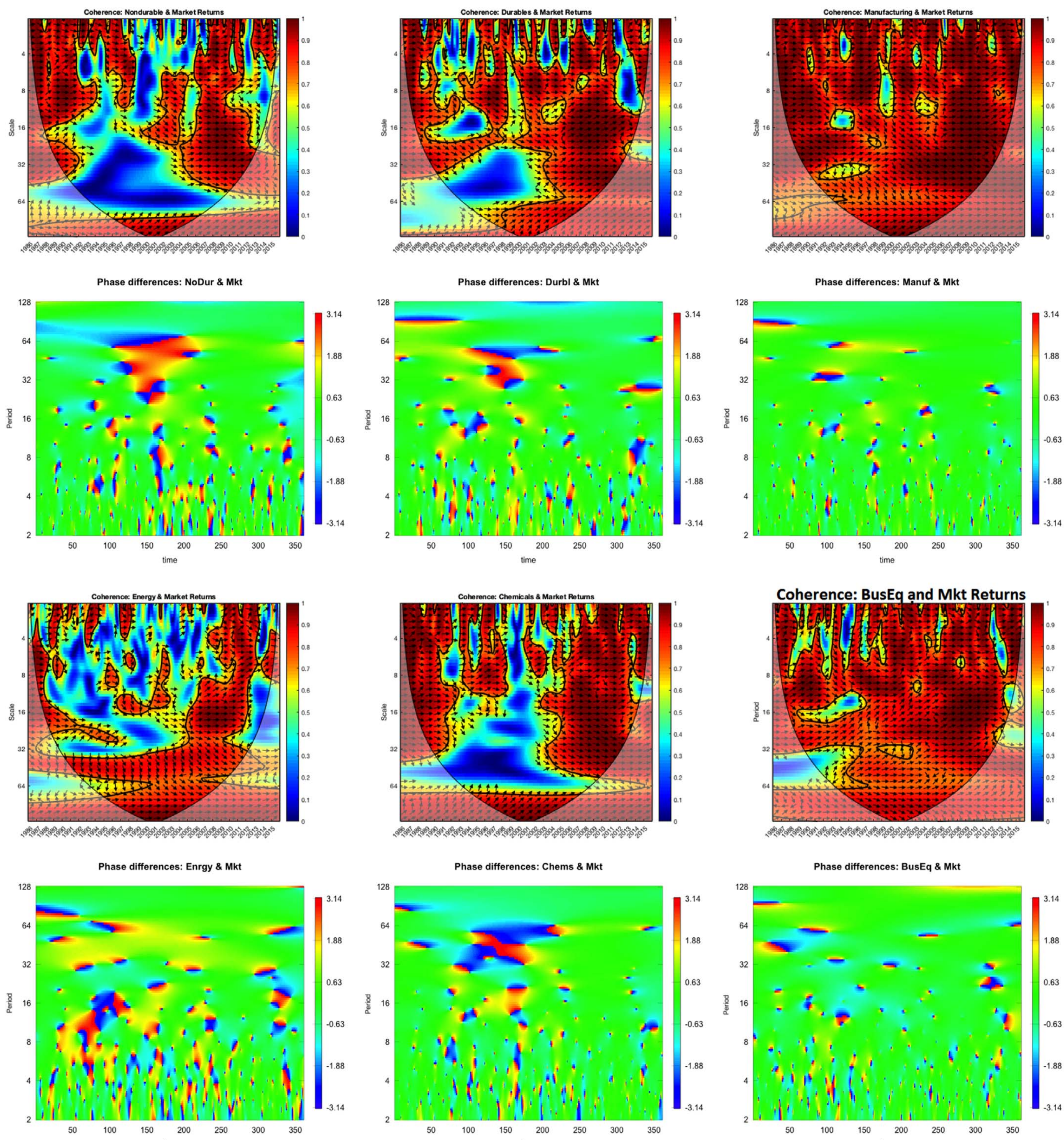

Fig. 4. Wavelet coherence and Phase Differences for Market returns with each sector. Note the inverted scale. Co-movement, of the market returns with the sector returns is lowest where the figure is blue. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

are statistically significant at the 95% level of confidence. The estimates of beta are all statistically significant at the 95% confidence level, and range from a low of 0.402 for Utils to a high of 1.363 for BusEq.

Variability in beta estimates over time is illustrated in Table 6 that contains summary statistics of estimated betas based on a 60-month rolling window. The variation in beta for a given sector, as measured by the maximum estimate less the minimum, ranges from a low of -0.006 for Utils and 0.275 for NoDur, to a high of 2.003 for BusEq.

Time plots of the rolling window beta estimates are presented below in Fig. 7. Several sectors, such as Shops and Manuf exhibit long stretches of stability. Other sectors such as Nodur and Hlth, vary widely over the sample period and follow a similar pattern in which the betas decrease until early 2005, and increase thereafter. Finally, the beta measure for Telcm jumps after the tech bubble bust in 2000 and continues to increase steadily until 2005, while beta for Money jumps in 2008 and remains high.

The fluctuations in the standard regression based estimates of beta are consistent with the changes and large breaks in coherence

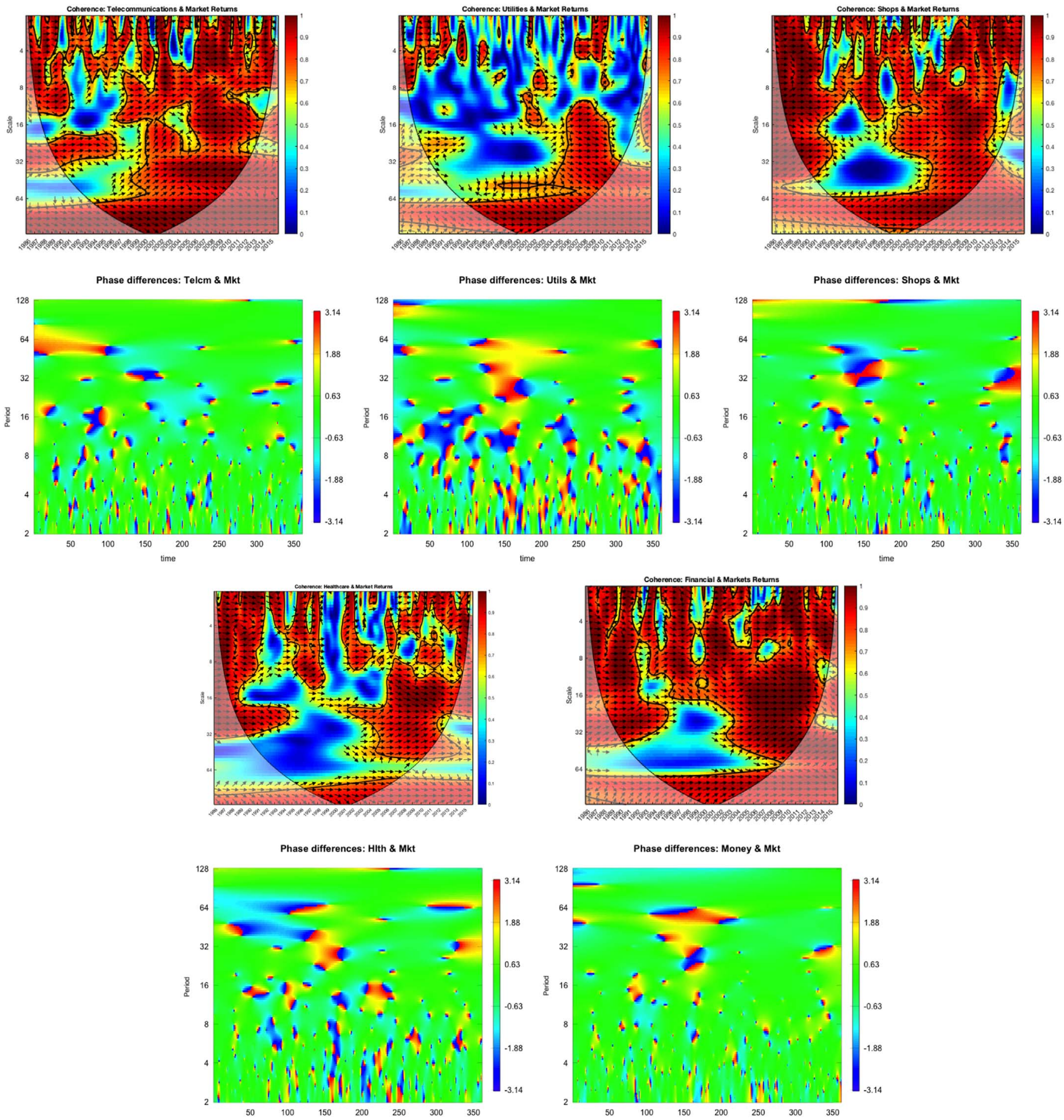

Fig. 5. Wavelet coherence and Phase Differences for Market returns with each sector. Note the inverted scale. Co-movement, of the market returns with the sector returns is lowest where the figure is blue. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

shown in the time/frequency plots of the continuous wavelet coherence (Fig. 5), that shows large breaks in coherence across time, even at low frequencies. So far, our analysis finds both large fluctuations in standard beta and large breaks in coherence over time. We next proceed to estimate scale betas for the sectors and investigate how the well-known strength of the wavelet in capturing time-varying behavior at different frequencies affects estimates of market risk.

4.7. Scale beta

The consumption capital asset pricing model has been used as the theoretical foundation in previous empirical studies that allows for changes in systematic risk over time. The consumption capital asset pricing model has been employed by Gencay, et al. (2001, 2003) to derive an estimate of systematic beta using wavelet methods. We assume that investors satisfy all their future consumption needs from security returns. This implies that

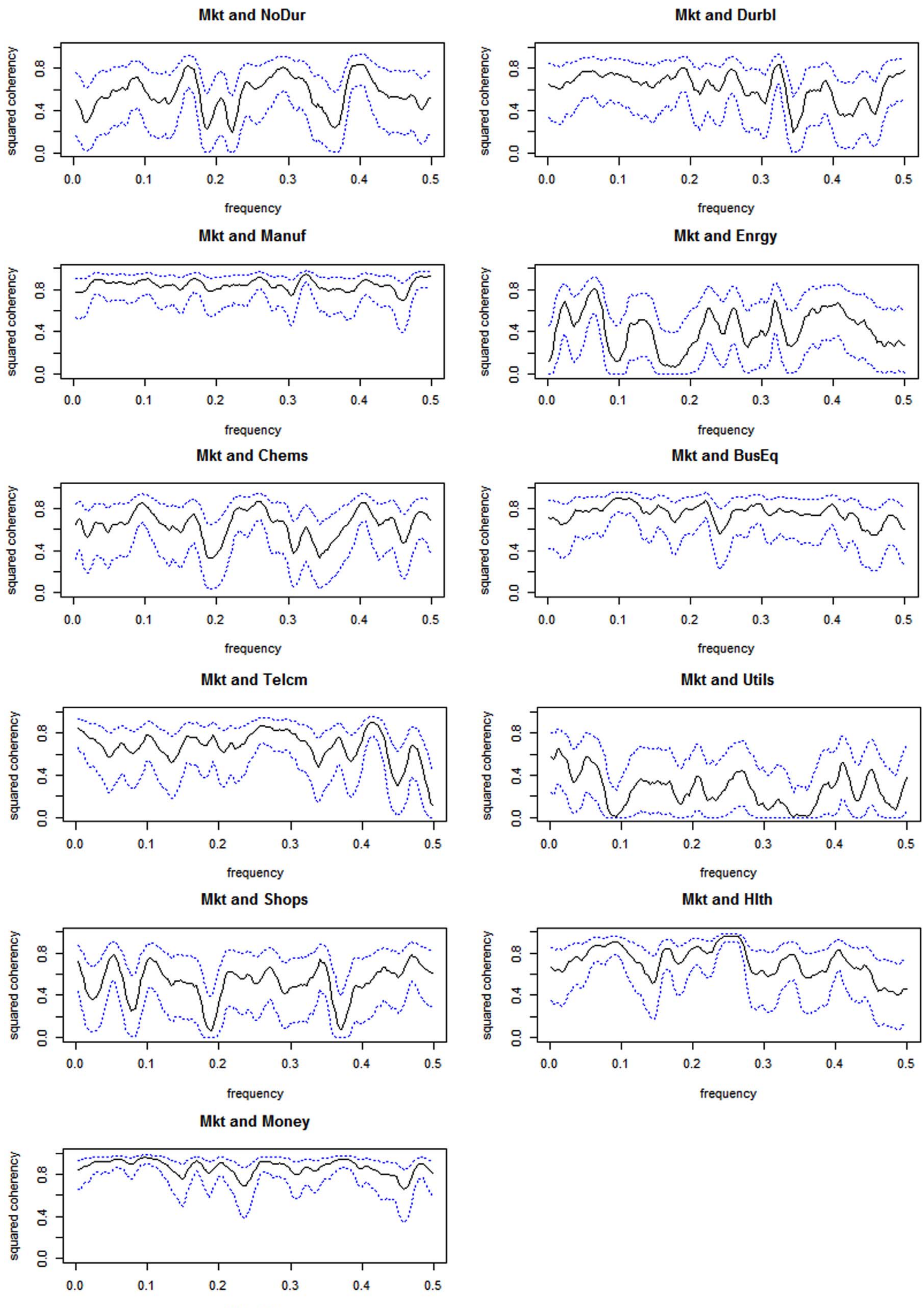

Fig. 6. Spectral analysis coherence of market returns with returns of each sector.

there are no non-investment sources of income and that the market portfolio is a proxy for total consumption.

The discrete wavelet transform is a useful tool for estimating the CAPM beta at different scales. We use a variation of the discrete wavelet transform known as the maximal overlap discrete wavelet transform, or MODWT to decompose each time series of returns into 6 scale series. The scale level is justified by the presence in the market of agents with strategies that differ by time scale. Explanations include

Table 5
Market model estimates, monthly returns Oct.1986-Oct.2016.

	Alpha	SE(alpha)	Beta	SE(beta)	Adj. R-Sq
NoDur	0.376	0.140	0.690	0.031	0.576
Durbl	−0.221	0.227	1.238	0.051	0.623
Manuf	0.101	0.116	1.114	0.026	0.837
Enrgy	0.244	0.227	0.738	0.051	0.368
Chems	0.201	0.142	0.828	0.032	0.654
BusEq	−0.077	0.192	1.363	0.043	0.736
Telcm	0.014	0.161	0.922	0.036	0.644
Utils	0.325	0.186	0.402	0.042	0.204
Shops	0.141	0.135	0.934	0.030	0.728
Hlth	0.339	0.167	0.756	0.374	0.531
Money	−0.007	0.155	1.063	0.035	0.722

Table 6
Beta estimates, rolling 60 month window, Oct. 1986-Oct.2016.

	NoDur	Durbl	Manuf	Enrgy	Chems	Buseq
Mean	0.685	1.221	1.109	0.730	0.808	1.384
Std Dev	0.221	0.326	0.159	0.233	0.212	0.312
Min	0.275	0.761	0.873	0.415	0.358	1.004
Max	1.071	1.768	1.376	1.261	1.105	2.022
	Telcm	**Utils**	**Shops**	**Hth**	**Money**	
Mean	0.940	0.421	0.916	0.756	1.091	
Std Dev	0.133	0.176	0.150	0.249	0.188	
Min	0.641	−0.006	0.720	0.328	0.693	
Max	1.350	0.719	1.256	1.242	1.391	

the assumption found in Gencay et al. (2003), that investors make their consumption/savings decisions at different times. We simply need for each time-scale the presence of investors with an interest in the market risk exposure they face for that time scale. Reasons may include differences in liquidity needs, shocks to wealth, and financial advice. We decompose the variance of the returns into a set of scale level variances by calculating the variance of the wavelet coefficients. The wavelet variance at scale j is the variance of the wavelet coefficients at that scale. Given a scale $\lambda_i = 2^{j-1}$, an unbiased estimate of the wavelet variance based on the MODWT is,

$$\sigma^2(\lambda_i) = \frac{1}{\widetilde{N}_j} \sum_{t=L_j-1}^{N-1} [\widetilde{d}_{j,t}]^2 \tag{10}$$

Where $\widetilde{d}_{j,t}$ is the MODWT wavelet coefficient at scale λ^j, $L_j = (2^j - 1)(L - 1)$ is the length of the scale λ_j wavelet filter, and $\widetilde{N}_j = N - L_j + 1$ is the number of coefficients not effected by a boundary, and $N = 2^J$. L_j=length of the j^{th} level wavelet filter.

Similarly the wavelet covariance at scale λ_j is:

$$\sigma_{xy}(\lambda_j) = \frac{1}{\widetilde{N}_j} \sum_{t=L_j-1}^{N-1} [\widetilde{d}^X_{j,t} \widetilde{d}^Y_{j,t}] \tag{11}$$

Following Gencay et al. (2003) we calculate scale level beta for sector i as

$$\beta_{ij} = \frac{cov(\widetilde{d}^m_{jt}, \widetilde{d}^i_{jt})}{\sigma_i^2(\lambda_j)} \tag{12}$$

Where $\widetilde{d}^m_{jt}$ is the vector of detail coefficients for the market variable at scale λ_j, and $\widetilde{d}^i_{jt}$ is the vector of detail coefficients for sector i.

Estimates of the scale betas using monthly sector returns are presented in Table 5 (for ease of comparison with also provide estimates of the standard betas). Asterisks (1, 2, or 3) next to a parameter estimate indicates that the scale beta is significantly different from the standard market beta at the 90%, 95%, or 99% level of confidence. While none of the scale estimates differ statistically from the standard beta measure at scale one (2-4 months) this is consistent with our finding of low coherence of sectors with the market at high frequencies. One interpretation of this finding is that at high frequencies, changes are short-lived and diversifiable. When it comes to sectors, short-term, transitory market movements do not produce scale betas that are significantly different from standard betas. One implication is that the short-run disaggregation of information captured by scale betas at the lowest scales is not of use to investors who care about the customized market risk exposure associated with investments in sectors. Although Shops has a scale two beta (4-8 months) that is statistically different from the standard beta measure our speculation is that the importance of discretionary income in this sector makes the typical consumer sensitive to market changes even at low scales. For example, unlike food, the consumer may postpone shopping trips to the mall or repair work, and such actions where consumption delays are possible may account for the sensitivity between sector returns and

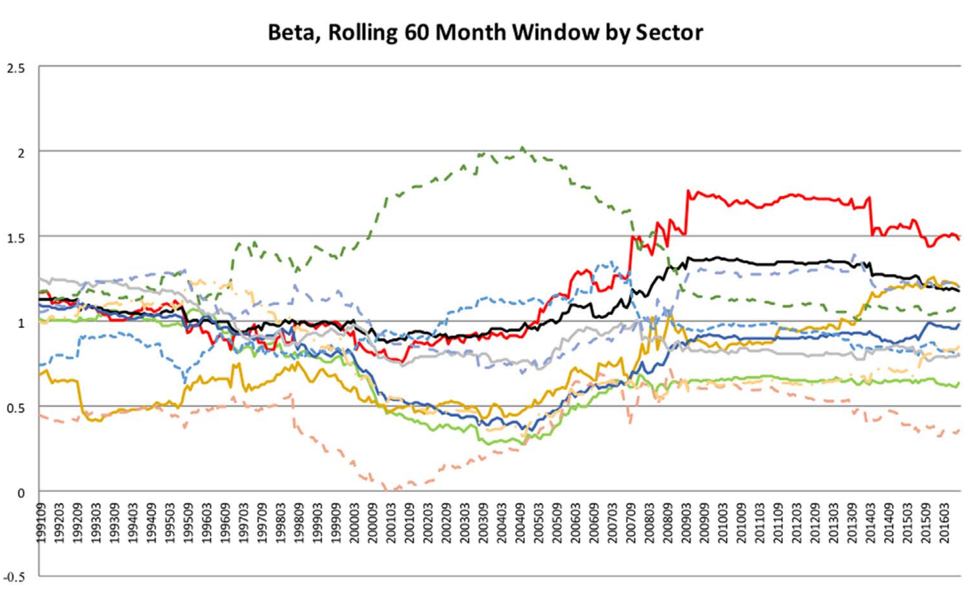

Fig. 7. Rolling beta 60 month window - sectors.

Table 7
Wavelet betas by sector, monthly returns, D(4) filter.

Scale	NoDur	Beta	SE(beta)	Adj. R-Sq	diff stat.		Telcm	Beta	SE(beta)	Adj. R-Sq	diff stat.	
Returns		0.690	0.031	0.576				0.922	0.036	0.644		
1		0.700	0.031	0.582	0.219			0.940	0.037	0.638	0.342	
2		0.723	0.031	0.601	0.742			0.904	0.033	0.679	−0.384	
3		0.690	0.029	0.605	−0.014			0.787	0.033	0.606	**−2.739**	***
4		0.689	0.032	0.567	−0.032			0.798	0.037	0.567	**−2.422**	***
5		0.529	0.035	0.381	**−3.417**	***		1.155	0.041	0.692	**4.283**	***
6		0.397	0.032	0.298	**−6.563**	***		1.402	0.038	0.793	**9.182**	***

Scale	Durbl	Beta	SE(beta)	Adj. R–Sq	diff stat.		Utils	Beta	SE(beta)	Adj. R–Sq	diff stat.	
Returns		1.238	0.051	0.623				0.402	0.042	0.204		
1		1.152	0.054	0.555	−1.154			0.367	0.044	0.157	−0.576	
2		1.276	0.046	0.678	0.559			0.412	0.040	0.225	0.177	
3		1.376	0.047	0.702	**1.993**	**		0.335	0.038	0.178	−1.190	
4		1.383	0.044	0.733	**2.165**	***		0.566	0.040	0.359	**2.854**	***
5		1.503	0.059	0.644	**3.418**	***		0.603	0.043	0.348	**3.341**	***
6		0.889	0.046	0.509	**−5.092**	***		0.701	0.024	0.704	**6.220**	***

Scale	Manuf	Beta	SE(beta)	Adj. R–Sq	diff stat.		Shops	Beta	SE(beta)	Adj. R–Sq	diff stat.	
Returns		1.114	0.026	0.837				0.934	0.030	0.728		
1		1.115	0.028	0.820	0.035			0.902	0.031	0.706	−0.739	
2		1.120	0.023	0.863	0.171			1.006	0.029	0.764	1.719	*
3		1.159	0.024	0.862	1.269			1.026	0.030	0.763	**2.172**	**
4		1.171	0.025	0.859	1.568			0.875	0.029	0.713	−1.404	
5		1.142	0.026	0.845	0.750			0.655	0.027	0.627	**−6.930**	***
6		0.783	0.021	0.797	**−9.977**	***		0.806	0.021	0.801	**−3.475**	***

Scale	Enrgy	Beta	SE(beta)	Adj. R–Sq	diff stat.		Hlth	Beta	SE(beta)	Adj. R–Sq	diff stat.	
Returns		0.738	0.051	0.368				0.756	0.037	0.531		
1		0.766	0.054	0.357	0.372			0.770	0.040	0.505	0.254	
2		0.688	0.050	0.343	−0.705			0.758	0.036	0.549	0.037	
3		0.756	0.048	0.405	0.259			0.736	0.033	0.582	−0.410	
4		0.853	0.040	0.559	1.781	*		0.718	0.031	0.598	−0.788	
5		0.742	0.048	0.399	0.063			0.597	0.039	0.393	**−2.943**	***
6		0.798	0.035	0.595	0.970			0.713	0.038	0.491	−0.807	

Scale	Chems	Beta	SE(beta)	Adj. R–Sq	diff stat.		Money	Beta	SE(beta)	Adj. R–Sq	diff stat.	
Returns		0.828	0.032	0.654				1.063	0.035	0.722		
1		0.848	0.033	0.643	0.424			1.002	0.037	0.665	−1.186	
2		0.837	0.031	0.675	0.195			1.120	0.030	0.797	1.259	
3		0.835	0.029	0.699	0.148			1.104	0.031	0.779	0.880	
4		0.842	0.033	0.647	0.292			1.143	0.033	0.769	1.674	*
5		0.739	0.032	0.590	**−1.972**	***		1.088	0.041	0.657	0.466	
6		0.497	0.027	0.491	**−7.998**	***		0.979	0.035	0.690	−1.703	*

Scale	BusEq	Beta	SE(beta)	Adj. R–Sq	diff stat.
Returns		1.363	0.043	0.736	
1		1.400	0.047	0.715	0.584
2		1.319	0.039	0.761	−0.767
3		1.383	0.037	0.794	0.353
4		1.277	0.040	0.742	−1.470
5		1.323	0.049	0.666	−0.620
6		1.265	0.045	0.687	−1.580

market returns at higher frequencies for this sector. However, additional research is needed to more precisely identify the factors that account for either high or low beta sectors at a specific scale.

At medium and high scales a very different picture emerges, ten of the eleven sectors have at least one scale beta that is significantly different from standard measure of beta. Such differences are consistent with the coherence plots. A significant difference between standard regression estimates of beta and scale betas is consistent with the explanation that frequency specific information that flows through a sector at different speeds and intensities results in breaks in coherence with the market over specific times and scales. For example, consider Telecom, a sector with four scale betas of significance. The coherence plot shows many more breaks or changes in color over medium or high scale than compared with BusEq. The coherence plot for BusEq shows a more solid color (red) over medium and high scales. We also find that scale betas do not increase or decrease monotonically for any of the sectors. There are also sectors such as Telecom that switch from low beta sector using the standard estimate to high beta sector at high scales, and a sector Manuf that is high beta based on standard beta estimate and low using high scale beta estimates.

One interpretation of the results where we find different estimates of market risk at the scale level but do not find any simple pattern across scales such as less risk at high scales is rooted in the underlying reality that the synchronization of sector returns and market returns that will produce significantly different scale betas depends on unique information that is captured at each horizon that affects the diversifica-

Fig. 8. Rolling Beta, 60 Day Window for Sectors by Scale: Comparison of standard beta with Scale 3 and Scale 4 betas over the sample period.

tion of sector risk. It would be implausible to find that every piece of unique information associated with each scale has effects on the diversification of market risk at the sector level, and therefore, not every scale beta is significantly different from standard. We also find that when comparing scale betas of significance there is not a monotonic relationship such as less market risk or greater market risk as the scale increases. In the Business Equipment sector there are no scales with significantly different estimates of market betas compared to the standard model estimate. This finding is consistent with the high coherence with the market over time and scale as illustrated in the coherence plot and reflected in the high beta of this sector both by standard estimates and over the six estimated scales. Again, pointing to the importance of differences in coherence across frequency that we do not see in the BusEq sector as an explanation for when horizon effects matter and produce scale betas that are significantly different from the standard estimate. Table 7.

4.8. Scale betas over time

To illustrate the importance of scale for estimates of beta, we use a rolling 60-month window over the sample period. Table A1 in the Appendix provides summary statistics of scale betas based on a rolling 60-month window for all eleven sectors. Fig. 8 shows the 60-month rolling window scale betas and the rolling window of the standard beta for four sectors that were found to have significantly different scale 3 and scale 4 betas. Fig. 8 illustrates that the scale betas may remain substantially above or below the standard beta for several years. This variability illustrates that adjusting for time variability through rolling window estimates without capturing time-varying behavior at different frequencies may lead to estimates of beta that do not accurately capture market risk dynamics. For example, consider a sector rotation strategy that is a popular strategy among active portfolio managers. This strategy typically recommends that investors overweight low beta sectors such as the Utilities and Telecommunications during a recession. Since proper identification of low beta sectors is crucial for deciding when to shift toward a more defensive stance the success of this strategy depends critically on having an accurate measure of beta.

5. Conclusions

Our exploration of the market risk of sectors finds that low scale betas are not significantly different from standard estimates. This is compatible with the interpretation that some information washes out in the range of frequencies associated with lowest scale and therefore, does not matter for estimates of market beta. It appears that the underlying economic relationships associated with estimates of market risk are not significantly changed at high frequency. Further research is needed to determine whether behavioral biases are operating at the lowest scales that produce transitory effects that evaporate when cooler heads prevail. A different story emerges for market moves that play out over medium and high scales. Differences in coherence across frequencies at medium and high scales matter for estimates of market risk for sectors. Our results show that when assessing the market risk of sectors simple patterns of increasing or decreasing systematic risk are not present. Comparing estimated standard beta coefficients of sectors with scale betas that were found to be significantly different our results find there are three sectors, Nodur, Manuf, and Hlth that have lower scale betas at medium and high scales than the standard estimates, and a sector, Utils with a higher scale betas compared to standard estimate.

Our main conclusion is that the importance of frequency specific information does not remain stable over time and therefore, a complete description of the systematic risk of investments in sectors requires estimates that capture time-varying behavior at different frequencies. Theoretical support for the existence of different types of market participants found in a wide body of research provides support for the application of a methodology that captures multi-period horizons. Allowing diversity into the investor universe does not always imply that horizon effects matter. While horizon effects are not found for any of the sectors at the lowest scales, horizon based effects are found for most sectors at medium and high scales. Investors with customized market exposure associated with sector focused investments who may not know a priori the time scale of their investments or those who know for sure that they are it for the long haul, will benefit from capturing the market risk of sector investments by broadening the definition of the beta heuristic to allow for scale effects.

Appendix

A1 A2.

Table A1
60 Month Rolling Window Betas by Sector and Scale.

	NoDur	Durbl	Manuf	Enrgy	Chems	Buseq
Mean	0.685	1.221	1.109	0.730	0.808	1.384
Std Dev	0.221	0.325	0.158	0.232	0.212	0.312
Skewness	0.071	0.421	0.423	0.690	−0.676	0.690
Kurtosis	−0.787	−1.394	−1.306	−0.541	−0.862	−1.004
Minimum	0.275	0.761	0.873	0.415	0.358	1.004
Maximum	1.071	1.768	1.376	1.261	1.105	2.022
	Tecm	**Utils**	**Shops**	**Hth**	**Money**	
Mean	0.940	0.421	0.916	0.756	1.091	
Std Dev	0.133	0.176	0.150	0.249	0.188	
Skewness	0.903	−0.752	1.058	0.427	−0.521	
Kurtosis	0.522	−0.316	−0.165	−1.044	−1.070	
Minimum	0.641	−0.006	0.720	0.328	0.693	
Maximum	1.350	0.719	1.256	1.242	1.391	

Table A2
Wavelet betas by sector, monthly returns, LA(8) filter.

Scale	NoDur	Beta	SE(beta)	Adj. R−Sq	diff stat.		Telcm	Beta	SE(beta)	Adj. R−Sq	diff stat.	
Returns		0.69	0.0312	0.5758				0.92	0.0361	0.6443		
1		0.7	0.031	0.5837	0.157			0.95	0.0375	0.6391	0.474	
2		0.73	0.0316	0.6007	1.009			0.9	0.0325	0.6819	−0.388	
3		0.67	0.0285	0.6082	−0.380			0.77	0.0328	0.605	**−3.097**	***
4		0.7	0.0316	0.5753	0.211			0.77	0.0362	0.5545	**−3.047**	***
5		0.51	0.0368	0.3495	**−3.679**	***		1.17	0.0419	0.6834	**4.444**	***
6		0.4	0.0313	0.313	**−6.506**	***		1.43	0.0352	0.8203	**10.001**	***
Scale	**Durbl**	**Beta**	**SE(beta)**	**Adj. R−Sq**	**diff stat.**		**Utils**	**Beta**	**SE(beta)**	**Adj. R−Sq**	**diff stat.**	
Returns		1.24	0.0507	0.623				0.4	0.0417	0.2036		
1		1.15	0.0549	0.5477	−1.193			0.36	0.0447	0.1516	−0.668	
2		1.26	0.0449	0.6869	0.364			0.43	0.04	0.2409	0.468	
3		1.42	0.0477	0.71	**2.569**	**		0.31	0.0366	0.1611	**−1.716**	*
4		1.35	0.0418	0.7439	**1.722**	*		0.58	0.0397	0.3699	**3.070**	***
5		1.59	0.0612	0.6511	**4.397**	***		0.61	0.045	0.3372	**3.407**	***
6		0.84	0.0433	0.5124	**−5.920**	***		0.74	0.0214	0.77	**7.257**	***
Scale	**Manuf**	**Beta**	**SE(beta)**	**Adj. R−Sq**	**diff stat.**		**Shops**	**Beta**	**SE(beta)**	**Adj. R−Sq**	**diff stat.**	
Returns		1.11	0.0259	0.8371				0.93	0.0301	0.7278		
1		1.12	0.0278	0.8175	0.065			0.9	0.0306	0.7058	−0.828	
2		1.11	0.023	0.867	−0.033			1.01	0.0296	0.7629	**1.763**	*

(continued on next page)

Table A2 *(continued)*

Scale	NoDur	Beta	SE(beta)	Adj. R–Sq	diff stat.		Telcm	Beta	SE(beta)	Adj. R–Sq	diff stat.	
3		1.17	0.0246	0.8636	**1.647**	*		1.04	0.0302	0.7681	**2.540**	**
4		1.16	0.0248	0.8587	1.290			0.88	0.0286	0.7229	−1.392	
5		1.18	0.0261	0.8496	**1.774**	*		0.6	0.0261	0.5907	**−8.498**	***
6		0.77	0.0196	0.8106 run	**−10.625**	***		0.82	0.0206	0.8135	**−3.262**	***
Scale	**Enrgy**	**Beta**	**SE(beta)**	**Adj. R–Sq**	**diff stat.**		**Hlth**	**Beta**	**SE(beta)**	**Adj. R–Sq**	**diff stat.**	
Returns		0.74	0.0508	0.3684				0.76	0.0374	0.5308		
1		0.77	0.0538	0.3593	0.387			0.77	0.0402	0.502	0.185	
2		0.69	0.0506	0.3366	−0.735			0.77	0.0366	0.5493	0.216	
3		0.76	0.0484	0.4046	0.287			0.73	0.0322	0.5843	−0.615	
4		0.85	0.0374	0.5867	**1.711**	*		0.74	0.0293	0.6364	−0.419	
5		0.75	0.0498	0.3835	0.119			0.57	0.0401	0.3581	**−3.410**	***
6		0.85	0.032	0.6619	**1.858**	*		0.71	0.0372	0.4994	−0.938	
Scale	**Chems**	**Beta**	**SE(beta)**	**Adj. R–Sq**	**diff stat.**		**Money**	**Beta**	**SE(beta)**	**Adj. R–Sq**	**diff stat.**	
Returns		0.83	0.0318	0.6535				1.06	0.0347	0.7224		
1		0.84	0.0334	0.6403	0.356			0.99	0.0377	0.6589	−1.338	
2		0.84	0.0308	0.6741	0.303			1.13	0.0294	0.8025	1.402	
3		0.84	0.028	0.7128	0.214			1.11	0.0304	0.7877	1.085	
4		0.84	0.0327	0.6488	0.340			1.15	0.0325	0.7752	**1.741**	*
5		0.74	0.0334	0.5765	**−1.928**	**		1.09	0.0443	0.6264	0.484	
6		0.5	0.0261	0.5007	**−8.099**	***		0.98	0.0319	0.7236	**−1.780**	*
Scale	**BusEq**	**Beta**	**SE(beta)**	**Adj. R–Sq**	**diff stat.**							
Returns		1.36	0.043	0.7365								
1		1.41	0.0468	0.7153	0.706							
2		1.3	0.0388	0.7585	−1.007							
3		1.39	0.0362	0.8037	0.465							
4		1.28	0.0387	0.7506	−1.519							
5		1.34	0.0524	0.6444	−0.354							
6		1.2	0.0422	0.691	**−2.722**	***						

References

Bali, Turan G., 2008. The intertemporal relation between expected returns and risk. J. Financ. Econ. 87.1, 101–131.

Bali, Turan G., Robert, F.Engle, 2010. The intertemporal capital asset pricing model with dynamic conditional correlations. J. Monet. Econ. 57.2, 377–390.

Bali, Turan G. and Robert F. Engle, 2012. The conditional CAPM explains the value premium. Working paper, Georgetown McDonough School of Business.

Bali, Turan G., Robert F. Engle, and Tang Yi, 2016. Dynamic conditional beta is alive and well in the cross-section of daily stock returns. Working Paper, Georgetown McDonough School of Business.

Baruník, Jozef, Lukáš Vácha, and Ladislav Krištoufek, 2011. Comovement of Central European stock markets using wavelet coherence: evidence from high-frequency data. Working Paper, IES.

Bekiros, S., Marcellino, M., 2013. The multiscale causal dynamics of foreign exchange markets. J. Int. Money Financ. 33, 282–305.

Bekiros, S., Nguyen, D.K., Uddin, G.S., Sjo, B., 2016. On the time-scale behavior of equity-commodity links: implications for portfolio management. J. Financ. Mark. Inst. Money 41, 30–46.

Bollerslev, Tim, Sophia Zhengzi Li, and Viktor Todorov, 2016. Roughing up beta: Continuous versus discontinuous beta and the cross-section of expected stock returns. Journal of Financial Economics, *120.3, 464-490.*

Bollerslev, Tim, Robert, F.Engle, Wooldridge, Jeffrey M., 1988. A Capital Asset Pricing Model with time-varying covariances. J. Political Econ. 96.1, 116–131.

Conlon, Thomas, Cotter, John, 2011. An empirical analysis of dynamic multiscale hedging using wavelet decomposition. J. Fut. Mark. J. Fut. Mark. 32.3, 272–299.

Fama, Eugene F., Kenneth, R.French, 2004. The capital asset pricing model: theory and evidence. J. Econ. Perspect. 18.3, 25–46.

Fernandez, Viviana., 2007. Multi-period Hedge Ratios for a Multi-asset Portfolio. When Accounting for Returns Co-movement. J. Fut. Mark. J. Fut. Mark. 28.2, 182–207.

Fernandez, Viviana. 2005. The International CAPM and a Wavelet-Based Decomposition of Value at Risk. Studies in Nonlinear Dynamics & Econometrics Article 4 ser. 9.1.

Gencay, Ramazan, Selcuk, Faruk, Whitcher, Brandon, 2003. Systematic risk and timescales. Quant. Financ. 3, 108–116.

Gencay, Ramazan, Selcuk, Faruk, Whitcher, Brandon, 2005. Multiscale systematic risk. J. Int. Money Financ. 24, 55–70.

Gencay, Ramazan, Faruk Selcuk, and Brandon Whitcher, 2010. An Introduction to Wavelets and Other Filtering Methods in Finance and Economics. (Academic Press, New York).

Graham, M., Kiviaho, J., Nikkinen, J., Omran, M., 2013. Global and regional co-movement of the MENA stock markets. J. Econ. Bus. 65, 86–100.

Handa, Puneet, Kothari, S.P., Wasley, Charles, 1989. The relation between the return interval and betas: implications for the size effect. J. Financ. Econ. 23.1, 79–100.

Harvey, C.R., 1989. Time-varying conditional covariances in tests of asset pricing models,? J. Financ. Econ. 24, 289–317.

Huang, P., James Hueng, C., 2008. Conditional risk-return relationship in a time-varying beta model. Quant. Financ. 8 (4), 381–390.

Jagannathan, Ravi, Wang, Zhenyu, 1996. The conditional CAPM and the cross-section of expected returns. J. Financ. 51 (1), 3–53.

Kamara, Avraham and Korajczyk, Robert A. and Lou, Xiaoxia and Sadka, Ronnie, Horizon Pricing (April 11, 2015).Journal of Financial and Quantitative Analysis (JFQA), Forthcoming. Available at SSRN: http://dx.doi.org/10.2139/ssrn.2114987.

Kahneman, Daniel, 2011. Thinking, Fast and Slow. Farrar, Straus & Giroux, New York.

Kriechbaumer, Thomas, Angus, Andrew, Parsons, David, Casado, Monica Rivas, 2014. An improved wavelet-ARIMA approach for forecasting metal prices. Resour. Policy 39, 32–41.

Khorana, Ajay, Nelling, Edward, 1997. The performance, risk and diversification of sector funds. Financ. Anal. J. 53.3, 62–74.

Levhari, David, Levy, Haim, 1977. The capital asset pricing model and the investment horizon. Rev. Econ. Stat. 59.1, 92–104.

Lewellen, J., Nagel, S., 2006. ?The conditional CAPM does not explain asset-pricing anomalies,?. J. Financ. Econ. 82, 289–314.

Lintner, John, 1965. Security prices, risk, and maximal gains from diversification. J. Financ. 20.4, 587–615.

Markowitz, Harry, 1952. Portfolio Selection. J. Financ. 7.1, 77–91.

Merton, Robert C., 1973. An intertemporal capital asset pricing model. Econometrica 41.5, 867–887.

Mossin, Jan, 1966. Equilibrium in a capital asset market. Econometrica 34.4, 768–783.

Noam, E.M., 2006. Fundamental Instability: why telecom is becoming a cyclical and oligopolistic industry. Inf. Econ. Policy 18, 272–284.

Ramsey, James, 2002. Wavelets in Economics and Finance: past and Future. Stud. Nonlinear Dyn. Econ. 6, 3, (Online).

Rua, António, Nunes, Luís C., 2009. International comovement of stock market returns: A wavelet analysis. J. Empir. Financ. 16 (4), 632–639.

Rua, António, Nunes, Luís C., 2012. A wavelet based assessment of market risk: The emerging markets case. Q. Rev. Econ. Financ. 52, 84–92.

Sharpe, William, 1964. Capital asset prices: a theory of market equilibrium under conditions of risk. J. Financ. 19 (3), 425–442.

Siegel, Jeremy J., 2005). The Future for Investors: Why the Tried and True Triumph over the Bold and New. Crown Business, New York.

Vacha, L., Barunik, J., 2012. Co-movement of Energy Commodities revisited: evidence from Wavelet Coherence Analysis. Energy Econ. 34(??), 241–247.

Wang, Yudong, Wu, Chongfeng, 2012. Long Memory in Energy Futures Markets: further Evidence. Resour. Policy 37.3, 261–272.